A Gradient-Descent-Based Null Space Planner for Dexterity Optimization during Upper-Limb Robot-Mediated Rehabilitation

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Introduction: Upper-limb motor impairment affects an estimated 50–75% of stroke survivors [1] and substantially diminishes independence and quality of life. Rehabilitation robots — particularly end-effector-based systems that avoid the need for complex exoskeletal fitting — offer a promising framework for delivering personalized, quantifiable upper-limb therapy while helping to address high costs, long wait times, and clinician scarcity [2,3,4]. Most existing platforms, however, are either planar or only permit limited ranges of motion [5], constraining therapy to limited configurations that cannot support the realistic task-specific practice thought to be necessary for effective rehabilitation [6]. To address this gap, we developed the OpenRobotRehab platform [7] shown in Figure 1 to enable training of motions in users' full 6-degree-of-freedom (DoF) end effector workspace. However, in practice, during large-workspace, multi-directional movements, this and other end-effector systems frequently encounter kinematic singularities. Near such configurations, manipulability degrades, leading to poor velocity transmission and reduced controllability that is particularly problematic for smooth free-space exercises. To address this limitation, we introduce a gradient-descent-based null-space velocity dexterity-optimization controller for the OpenRobotRehab platform [7]. The proposed method optimizes the Yoshikawa manipulability index [8] to enhance kinematic dexterity, while projecting the optimization into the null space of the velocity Jacobian to preserve the commanded end-effector pose. This approach reduces proximity to kinematic singularities and enables more reliable execution of natural 6-DoF upper-limb rehabilitation trajectories.

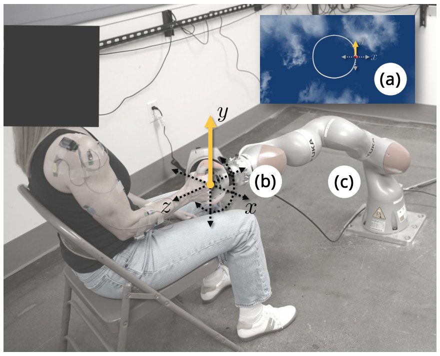

Figure 1: OpenRobotRehab [7] upper limb rehabilitation platform. Users follow displayed trajectories like circle (a) by transmitting forces and torques though a Bota Systems SensONE 6-axis force load cell (b) to move KUKA LBR iiwa 14 R820 7-DoF cobot (c).

Methods: In this work, we quantify dexterity using the Yoshikawa manipulability index $w = \sqrt{\det(JJ^\top)}$ for velocity Jacobian matrix J [8], which serves as the objective for both global trajectory optimization and local re-optimization. An offline configuration-space trajectory corresponding to the nominal rehabilitation path is obtained by minimizing $-w$ via CasADi-based gradient descent, subject to null-space projection to enforce task-space constraints [9]. The resulting trajectory is executed provided the RMS deviation from the desired end-effector path remains below a prescribed threshold; when exceeded, a null-space artificial potential field (APF) formulation of the manipulability gradient is activated [10]. At each configuration, the numerical Jacobian is evaluated, joint-space gradients of w are estimated, and a bounded update projected into the Jacobian null space is applied to increase manipulability without altering the end-effector pose. The controller, with and without APF augmentation, was evaluated in a simulation designed to emulate a human executing a three-dimensional circular motion while interacting with the end-effector of the OpenRobotRehab platform [7], representing a robot-assisted rehabilitation scenario. To emulate variability, the nominal smooth 2D circular trajectory was perturbed with Gaussian noise and intermittent large deviations, yielding a non-ideal 3D human-like motion profile. As a baseline, a naive inverse kinematics controller was implemented, using the waypoints of the simulated trajectory and a least squares approximation to generate joint inputs without dexterity optimization.

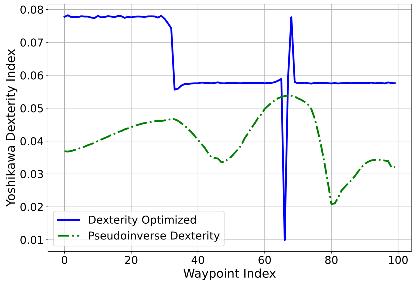

Figure 2: Comparison of the Yoshikawa dexterity index along a 100-point trajectory for naive (*green*) and dexterity-optimized (*blue*) control strategies. The dexterity-optimized controller maintains consistently higher manipulability than the naive pseudoinverse-based approach across almost all waypoints. A pronounced transient drop and spike near waypoint 65 indicates sensitivity to local Jacobian conditioning, after which dexterity returns to its nominal level.

Results & Discussion: During the simulated circular task, our proposed dexterity-optimizing controller largely produced appreciable improvement in scalar dexterity across the entirety of the task as compared with the naive inverse-kinematics-based controller, as illustrated in Figure 2. However, scalar optimization of the Yoshikawa index either using only offline optimization or in conjunction with the APF-augmented controller did not yield appreciable improvements in task-level rehabilitation performance. We attribute these limitations to a mismatch between scalar manipulability maximization and task-level functional performance: the offline optimizer implicitly assumed near-perfect tracking of the nominal path — an assumption that degrades in proportion to motor impairment — while the online APF-based null-space updates produced only marginal benefit before larger gains induced instability due to configuration-dependent Jacobian variation. Consequently, although scalar dexterity increased, singular configurations and task-relevant loss of manipulability during rehabilitation were not substantively mitigated. To address these limitations, future work will transition the formulation from a scalar manipulability metric to the velocity manipulability ellipsoid, which captures directional dexterity characteristics in task space. Building on this representation, we will develop an online gradient-descent-based null space controller that reshapes the velocity manipulability ellipsoid to increase dexterity along the anticipated direction of patient motion during task execution. We hypothesize that this directionally weighted velocity-based formulation will produce greater improvements in rehabilitation task performance, pending empirical validation.

Significance: As upper-limb robot-facilitated rehabilitation is investigated and employed with increasing frequency, it is essential to ensure that patients are able to perform the full range of free-space movements required for activities of daily living during therapy. Current systems rarely support such comprehensive motion and typically constrain users due to their limited dexterity and/or limited degrees of freedom. In this and future work, we aim to develop systems and algorithms that enable unimpeded human motion, thereby supporting more effective upper-limb rehabilitation.

References: [1] Anwer et al. (2022), *Healthcare*. [2] Moulaei et al. (2023), *Archives of Public Health*. [3] Wei et al. (2024), *BMC Neurology*. [4] Lin et al. (2015), *PM&R*. [5] Scibilia et al. (2024), *CASE*. [6] Maier et al. (2019), *Frontiers in Systems Neuroscience*. [7] Anand et al. (2025), *ICORR*. [8] Yoshikawa (1990), *Foundations of Robotics: Analysis and Control*. [9] Andersson et al. (2019), *Mathematical Programming Computation*. [10] Khatib (1986), *IJRR*.